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LETTER TO THE EDITOR

The exponent γ for the spin-$\frac{1}{2}$ Ising model on the face-centred cubic lattice

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Abstract. The high-temperature, zero-field susceptibility series of the spin-$\frac{1}{2}$ Ising model on the face centred cubic lattice is analysed assuming the asymptotic form $A_1t^{-\gamma}+A_2t^{-\gamma+1}+Bt^{-\gamma+\Delta_1}$ where t is the reduced temperature.

Good convergence is obtained for $\gamma=1{\cdot}241$ and $\Delta_1=0{\cdot}496$, the values predicted by renormalisation group theory. Attempts to fit the series coefficients with $\gamma=1{\cdot}25$ and $B=0$, do not prove as successful. The amplitudes A_2/A_1 and B/A_1 are estimated.

The high-temperature susceptibility χ_0 of the spin-$\frac{1}{2}$ Ising model has been the subject of much recent investigation. The interest is chiefly in the precise value of the exponent γ which characterises the divergence of χ_0 at the critical temperature. Series expansion studies (Domb and Sykes 1957, Fisher 1967, Sykes *et al* 1972) have consistently given the estimate

$$\gamma=1{\cdot}25\pm0{\cdot}003. \tag{1}$$

However, recent renormalisation group (RG) calculations predict a lower value of γ. le Guillou and Zinn Justin (1977) quote,

$$\gamma=1{\cdot}2402\pm0{\cdot}0009, \tag{2}$$

while Baker *et al* (1978), using a slightly modified calculation, obtain,

$$\gamma=1{\cdot}241\pm0{\cdot}004, \tag{3}$$

In addition, RG theory predicts the existence of an additive confluent correction term modifying the dominant singularity characterised by γ. Associated with this confluent singularity is another exponent Δ_1, which has been estimated by le Guillou and Zinn Justin (1977) to be

$$\Delta_1=0{\cdot}493\pm0{\cdot}007, \tag{4}$$

and by Baker *et al* (1978) as,

$$\Delta_1=0{\cdot}496\pm0{\cdot}004. \tag{5}$$

The first attempt at analysing susceptibility series for the more general asymptotic form,

$$\chi_0\sim A(t)t^{-\gamma}+Bt^{-\gamma+\Delta_1}.$$

$$(t=1-T_c/T), \tag{6}$$

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was that of Camp and Van Dyke (1975). They made a study of the general spin Ising model based on series expansions in powers of $K = J/kT$ to order K^{10}. They found evidence of the confluent singularity term $t^{-\gamma+\Delta_1}$, but concluded that the amplitude B vanishes for the spin-$\frac{1}{2}$ case ($S = \frac{1}{2}$). The exponent γ was estimated to be 1·25, and for $S > \frac{1}{2}$, Δ_1 was found to be $0{\cdot}5 \pm 0{\cdot}08$, in reasonable agreement with RG calculations.

In a recent study, Gaunt and Sykes (1979) adduced further evidence for the estimate $\gamma = 1{\cdot}25$ for the face centred cubic (FCC), body centred cubic (BCC), simple cubic (SC) and diamond (DIA) lattices, for $S = \frac{1}{2}$. Interference from the non-confluent antiferromagnetic singularity was reduced by an Euler transformation (Gaunt and Guttmann 1974) and the transformed series were used to estimate γ. However, their analysis did not explicitly take into account the possibility of a correction term of the form $t^{-\gamma+\Delta_1}$.

A more recent investigation into the presence of subdominant critical indices is that of Bessis *et al* (1979). Using a variant of the Baker–Hunter method of analysis for confluent singularities, they made a study of spin-$\frac{1}{2}$ susceptibility series for several lattices in two to six dimensions. In three dimensions, they estimated the dominant critical index γ as,

$$\gamma = 1{\cdot}2506 \pm 0{\cdot}0015. \qquad (7)$$

They also found evidence of a universal subdominant index γ_s, which was estimated at

$$\gamma_s = 0{\cdot}42 \pm 0{\cdot}11. \qquad (8)$$

The value of γ_s is clearly incompatible with the RG value of $\gamma - \Delta_1 = 0{\cdot}745 \pm 0{\cdot}008$ (Baker *et al* 1978). Bessis *et al* found that the three loose-packed lattices (BCC, SC and DIA) were specially amenable to their analysis while the FCC lattice gave the poorest convergence. Their estimates (7) and (8) were based on the loose-packed lattices only. For the FCC lattice, they quote,

$$\gamma = 1{\cdot}2458, \qquad \gamma_s = -0{\cdot}13. \qquad (9)$$

This estimate for γ is significantly lower than 1·25, but still higher than the RG prediction. γ_s however, is even further away from 0·745.

In this paper, we reanalyse the fifteen-term χ_0 series for the spin-$\frac{1}{2}$ Ising model on the face centred cubic lattice, assuming the asymptotic form (6). We find, contrary to the work of Camp and Van Dyke (1975), that the amplitude B does not vanish. We also find that while the series coefficients can be fitted reasonably well to $\gamma = 1{\cdot}25$, with B set to zero, convergence improves noticeably for $\gamma = 1{\cdot}241$, $\Delta_1 = 0{\cdot}496$, the RG predictions. Given the uncertainties inherent in any extrapolation procedure, it is not possible to assert with complete certainty that the RG values for γ and Δ_1 are correct. However, the analysis presented here, does, for the face centred cubic lattice at least, provide some support for the RG predictions.

Attempts to fit χ_0 for the loose-packed lattices did not prove very successful and we feel that this is due to the presence of the antiferromagnetic singularity at $-K_c$. Camp and Van Dyke (1975) reported similar difficulties. Their conclusions were based essentially on the FCC lattice, which they found was the best behaved.

Our analysis parallels that of Camp and Van Dyke (1975). To facilitate comparison with their work, we will use K as expansion variable rather than the more usual $v = \tan K$. (However, we have also carried out the analysis in v, and find no significant change in convergence.) Assuming the asymptotic form (6), we expand $A(t)$ in a Taylor series about $t=0$, retaining terms to first order in t. We obtain,

$$\chi_0=\sum_{n\geq 0} a_n K^n = A_1 t^{-\gamma}+A_2 t^{-\gamma+1}+Bt^{-\gamma+\Delta_1}. \tag{10}$$

We form the ratios $R_n = a_n/a_{n-1}$, which should behave, in the limit $n\to\infty$, as,

$$R_n = K_c^{-1}\left(1+\frac{\gamma-1}{n}+\frac{a}{n^{1+\Delta_1}}+\frac{b}{n^2}\right). \tag{11}$$

The amplitudes 'a' and 'b' are related to A_2 and B in the following manner,

$$-b = A_2/A_1[\Gamma(\gamma)/\Gamma(\gamma-1)],$$
$$-a = B/A_1[\Delta_1\Gamma(\gamma)/\Gamma(\gamma-\Delta_1)]. \tag{12}$$

We now fix γ at 1·25 and Δ_1 at 0·5 and use successive triplets of R_n to solve for K_c, a and b. The results are presented in table 1(a). We find that for $n = 10$, the values of 'a' and 'b' are $-0{\cdot}007$ and 0·003 respectively, in agreement with Camp and Van Dyke

Table 1(a). $\gamma = 1{\cdot}25$, $\Delta_1 = 0{\cdot}5$.

n	K_c^{-1}	a	b
9	9·793837	−0·00145	−0·00963
10	9·794406	−0·00729	+0·00307
11	9·794997	−0·01445	0·01951
12	9·795328	−0·01911	0·03079
13	9·795495	−0·02197	0·03756
14	9·795593	−0·02358	0·04228
15	9·795676	−0·02528	0·04695

Table 1(b). $\gamma = 1{\cdot}241$, $\Delta_1 = 0{\cdot}496$

n	K_c^{-1}	a	b
9	9·795658	0·03178	−0·04464
10	9·796027	0·02806	−0·03651
11	9·796457	0·02294	−0·02467
12	9·796657	0·02018	−0·01794
13	9·796713	0·01929	−0·01569
14	9·796718	0·01921	−0·01546
15	9·796721	0·01914	−0·01529

Table 1(c). $\gamma = 1{\cdot}25$, $a = 0$

n	K_c^{-1}	b
9	9·793695	−0·01278
10	9·793803	−0·01370
11	9·793969	−0·01543
12	9·794141	−0·01760
13	9·794229	−0·01998
14	9·794439	−0·02244
15	9·794564	−0·02499

(1975). But the estimates for 'a' are increasing quite rapidly in magnitude and show no signs of converging even at $n = 15$. Fixing Δ_1 at 0·496 (the central value in (5)) makes very little difference to the fit.

The situation changes when γ is set at 1·241. In table 1(b), we present estimates for K_c^{-1}, 'a' and 'b', calculated for $\gamma = 1{\cdot}241$, $\Delta_1 = 0{\cdot}496$. We find a noticeable improvement in convergence. The last three values of 'a' (n = 13, 14 and 15) remain constant to within 0·5%, and those of 'b' to within 1%. Small changes to $\Delta_1(\pm 0{\cdot}005)$, make no appreciable difference. We estimate,

$$\begin{aligned} K_c^{-1} &= 9{\cdot}7967 \pm 0{\cdot}0001 \\ a &= 0{\cdot}019 \pm 0{\cdot}001 \\ b &= -0{\cdot}015 \pm 0{\cdot}002. \end{aligned} \tag{13}$$

Using (9), we obtain,

$$\begin{aligned} A_2/A_1 &= 0{\cdot}062 \pm 0{\cdot}008 \\ B/A_1 &= -0{\cdot}052 \pm 0{\cdot}002. \end{aligned} \tag{14}$$

To make a direct comparison with the case where there is no correction term of the form $t^{-\gamma+\Delta_1}$, we give in table 1(c), estimates for K_c^{-1} and 'b', using $\gamma = 1{\cdot}25$ and 'a' set to zero. Again convergence is not as good as in 1(b).

The analysis given here seems to lend some support to the RG predictions for the values of the dominant and subdominant critical indices γ and Δ_1. Our analysis has, admittedly, been confined to the FCC lattice, but then, this lattice has always been considered to provide the best converged series for most thermodynamic properties.

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